Discrete Element Modelling of VACUUMATICS

Frank HUIJBEN
PhD Researcher
Structural Design
Eindhoven University of Technology (TU/e),
Eindhoven, the Netherlands
F.A.A.Huijben@tue.nl

Frank Huijben, born 1979, received his degree in architecture and structural engineering from the TU/e in 2008. Besides his PhD he works as a structural engineer at ABT Consulting Engineers in the Netherlands.

Frans VAN HERWIJNEN
Emeritus Professor
Structural Design,
Eindhoven University of Technology (TU/e)
Eindhoven, the Netherlands
F.v.Herwijnen@abt.eu

Frans van Herwijnen, born 1953, received his degree in structural engineering from the TU/e in 1978. He works as a consulting engineer at ABT Consulting Engineers in the Netherlands.

Rob NIJSSE
Professor Structural Engineering,
Delft University of Technology (TUD)
Delft, the Netherlands
R.Nijsse@tudelft.nl

Rob Nijsse, born 1953, received his degree in civil engineering from the TUD in 1979. He works as a structural consulting engineer at ABT Consulting Engineers in the Netherlands and as a professor in structural engineering at the TUD.

Summary

Vacuumatics consist of independent particles inside an airtight enclosed membrane, that are prestressed due to a difference in (air) pressure. Analytical and numerical research of vacuum prestressing has illustrated that the effective prestressing forces can be divided into two interrelated prestressing components. Due to the granular characteristics of Vacuumatics, the material behaviour can be modelled by means of the Discrete Element Method (DEM). The individual prestressing components acting on the edge particles of vacuumatic structures can be simulated by means of a specialised Atmospheric Pressure Model in HADES (by Habanera). Analytically defined equations form the basic input in this simulation process. These simulations enable us to analyse (visually as well as numerically) the prestressing forces, but also the contact forces and the displacements of each particle due to this vacuum prestressing. Furthermore, bending phenomena of beam-shaped Vacuumatics can be analysed in detail, providing us with insight to describe and predict the structural properties of any type of vacuumatic structure.

Keywords: *Vacuumatics, Discrete Element Modelling, vacuum prestressing, bending simulation.*

1. Introduction

Vacuumatic structures, or Vacuumatics, are based on a new and yet unexplored structural principle, as they consist of structural aggregates (particles) that are tightly packed inside a flexible membrane envelope (skin). The structural integrity is obtained by applying a (controllable) negative pressure, or partial vacuum, inside the enclosed skin, hence prestressing and stabilising the particles in their present configuration by means of the atmospheric (air) pressure. This process is referred to as 'vacuum prestressing'. The beneficial characteristics of Vacuumatics, to be 'freely' shaped and to be re-shaped repeatedly to new requirements, provide a promising approach for the design of a truly flexible and reconfigurable (self-supporting) concrete formwork system [1]. However, in order to effectively apply Vacuumatics as a (temporary) load-bearing structure in a controlled setting, the structural properties as well as the morphological boundary conditions need to be well defined.

The structural behaviour of Vacuumatics largely depends on the individual material characteristics of its components, the filler particles and the skin envelope, as well as on the interaction of the particles with the enclosing skin and the particles mutually. For this reason it is relatively complex to describe (and to predict) the structural properties of any vacuumatic structure.

In the Van Musschenbroek laboratory at the Eindhoven University of Technology (the Netherlands) a large number of four-point bending tests have been conducted, in order to investigate the flexural rigidity and the bending behaviour of beam-shaped vacuumatic structures with varying components (Fig.1). The systematic approach of this experimental research enabled us to determine the influence of several material parameters (e.g. particle size and skin elasticity) on the overall flexural rigidity in a qualitative manner [2].

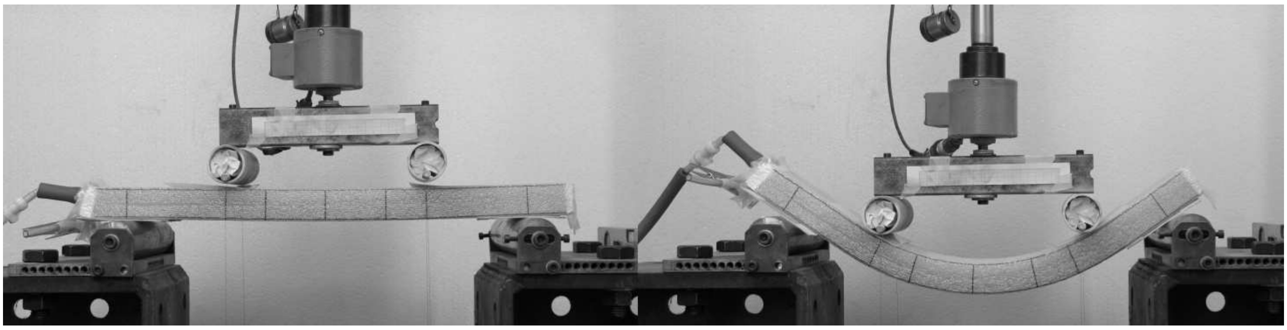

Fig. 1: bending test on a beam-shaped Vacuumatics specimen

2. Vacuum prestressing

The first step in analysing the behaviour of Vacuumatics is to determine how the vacuum prestressing can be described. For practical reasons the particles are represented as solid disks. The difference in (air) pressure, referred to as the (external) 'vacuum pressure', causes the flexible membrane envelope to be pushed into the cavities in between the edge particles, creating a line of perfectly circular curves with points of inflection at the intersections of the skin and the particles (Fig.2a). The part of a particle that is being covered by the membrane is directly subjected to the vacuum pressure. Furthermore, due to this external pressure, the piece of skin in between two adjacent particles tends to 'pull' these two bodies closer together (Fig.2b). This pulling action can be explained by the physics principle of surface tension. That is, the reaction forces of a curved membrane under a multidirectional pressure, in this case referred to as the 'skin forces', can easily be determined for each curvature (*i*) by means of the 2-dimensional representation of the Young-Laplace equation.

$$F_{s;i} = R_{s;i} P_v \tag{1}$$

$F_{s;i}$ *= skin force [N/mm*^{1}*],* $R_{s;i}$ *= radius of skin curvature [mm],* P_v *= vacuum pressure [MPa]*

As the above mentioned is the case for each edge particle, it can be understood that the entire collection of particles is being externally prestressed. This vacuum prestressing results in compression forces between particles that interconnect. The force distribution throughout the particle packing can be illustrated by drawing a grid from the centre points of the connecting particles (Fig.2c), which in case of regular close packing (with equally-sized particles) resembles a triangulated truss structure. The magnitude of the individual contact forces can then easily be derived by means of mechanics equilibrium and basic geometry.

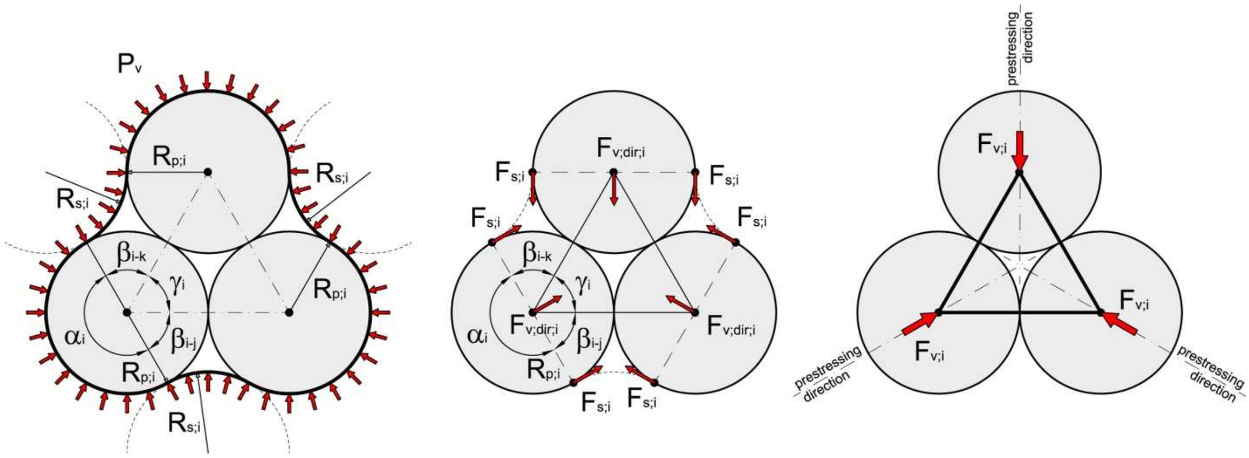

Fig. 2: vacuum pressure (a), prestressing components (b) and prestressing forces (c)

2.1 Prestressing force components

Analytical and numerical research on the basics of vacuum prestressing of Vacuumatics has illustrated that the total vacuum prestressing forces for each individual particle (i) in contact with the membrane (referred to as edge particles) can be divided into two interrelated components, referred to as the direct vacuum prestressing force and the indirect vacuum prestressing force [3]. In case of equally-sized (edge) particles and uniform skin radii these components can be determined with equations (2) and (3), see also Fig.2.

$$F_{v;dir;i} = 2R_{p;i}P_v \sin(\tfrac{1}{2}\alpha_i) \tag{2}$$

$$F_{v;indir;i} = 2R_{s;i}P_v \sin(\tfrac{1}{2}\alpha_i) \tag{3}$$

$$F_{v;i} = F_{v;dir;i} + F_{v;indir;i} \tag{4}$$

$F_{v;dir;i}$ = direct vacuum prestressing force [N/mm^1], $F_{v;indir;i}$ = indirect vacuum prestressing force [N/mm^1], $R_{p;i}$ = particle radius [mm], $R_{s;i}$ = skin radius [mm], P_v = vacuum pressure [MPa], α_i = covered angle [rad], $F_{v;i}$ = total vacuum prestressing force [N/mm^1]

In case of various-sized particles, a discontinuous range of outer particles, or a varying range of skin curvature radii, equation (3) is no longer valid (Fig.3a). For this equation to be valid, factor ($2R_{s;i}$) needs to be split up into two separate skin radii ($R_{s;i-j}$) and ($R_{s;i-k}$). Equation (2) remains unaltered for determining the direct vacuum prestressing force for each particle, considering the fact that the covered angle (α_i) can be defined as a function of the individual angles (γ_i), (β_{i-j}) and (β_{i-k}). These angles on their part can each be describes as a function of the particle distances (d_{i-j}, d_{i-k}, and d_{j-k}), the radii of skin curvature ($R_{s;i-j}$ and $R_{s;i-k}$) and the particle radii ($R_{p;i}$), by means of the law of cosines (which will not be elaborated in this paper). Therefore, when these variables are known, the indirect prestressing force can be calculated. The above-mentioned leads to the following equations.

$$F_{v;indir;i} = (R_{s;i-j} + R_{s;i-k})P_v \sin(\tfrac{1}{2}\alpha_i) \tag{5}$$

$$\alpha_i = 2\pi - \gamma_i - \beta_{i-j} - \beta_{i-k} \tag{6}$$

$F_{v;indir;i}$ = indirect vacuum prestressing force [N/mm^1], $R_{p;i}$ = particle radius [mm], $R_{s;i-j}$ = skin radius in between particle (i) and (j) [mm], $R_{s;i-k}$ = skin radius in between particle (i) and (k) [mm], P_v = vacuum pressure [MPa], α_i = covered angle [rad], γ_i = configuration angle of centre line of edge particles (i), (j) and (k) [rad], β_{i-j} = angle of skin centre and centre line of edge particles (i) and (j) [rad], β_{i-k} = angle of skin centre and centre line of edge particles (i) and (k) [rad]

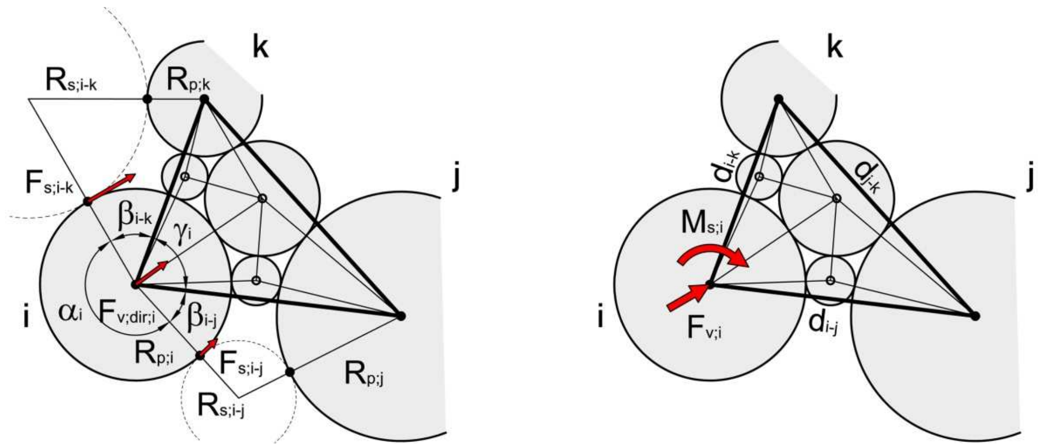

Fig. 3: unequal vacuum prestressing components (a) and rotating motion due to varying skin radii (b)

In addition, we must acknowledge the fact that different skin radii result in different skin forces, which might initiate a rotating motion of the particle in question (Fig.3b). This rotating motion can be defined as a moment of force, or torque ($M_{s;i}$), described by the acting skin forces and the moment arm, which in this case is the particle radius.

$$M_{s;i} = F_{s;i-k} R_{p;i} - F_{s;i-j} R_{p;i} = (F_{s;i-k} - F_{s;i-j}) R_{p;i} \tag{7}$$

$M_{s;i}$ = torque [Nmm^1], $R_{p;i}$ = particle radius [mm], $F_{s;i-j}$ = skin force in between particle (i) and (j) [N], $F_{s;i-k}$ = skin force in between particle (i) and (k) [N]

From equations (2) and (5), as well as Fig.2 and 3, it is clearly illustrated that the total vacuum prestressing force for each individual particle largely depends on the amount of skin coverage. In general a smaller total skin length (or skin perimeter), and thus a smaller covered angle and a larger skin radius, will lead to a higher vacuum prestressing of the structure. Therefore, assuming that the skin perimeter is at its minimum in its initial (pre-vacuumatic) state, the total vacuum prestressing largely depends on the elasticity of the membrane material.

3. Discrete Element Modelling

From a material point of view Vacuumatics can be regarded as a special type of granular material. The structural behaviour can therefore be analysed numerically by means of the Discrete Element Method (DEM), or Distinct Element Method. DEM differs from continuum methods in that the (material) behaviour of the entire material/mixture is simulated by considering the behaviour of each constituent in the mixture individually. By simulating the complex interactions between grains mutually and the influence of the environment on the individual grains, the behaviour of the whole mixture can be evaluated.

3.1 HADES

HAbanera's Discrete Element Simulator (HADES) is a discrete element software package, developed by Habanera [4], that simulates granular phenomena. In HADES the dynamics of the individual particles is evaluated by integrating Newton's second law of motion. The net force (F_i) and net torque (M_i) result from the summation over the individual forces (f_{ij}) and torques (m_{ij}) that act on body (i). The summation is over the number of actuators (j). The result of an active actuator on a particle is a force.

$$F_i = \sum_j f_{ij} \tag{8}$$

$$M_i = \sum_j m_{ij} \tag{9}$$

In HADES the force (f_{ij}) that acts on a particle (i) due to an actuator (j) is independent from the force (f_{ik}) that acts on the same body but from a different actuator (k). This means that each actuator can evaluate its influence on a group of particles, independently from any other actuators that may be active. For example, the evaluation of the gravity force can be performed independent from the evaluation of the drag force and contact force. In HADES these actuators are encapsulated in so called Models. Each Model calculates its addition to the total force that acts on each particle. The user can specify in an input file which Models (actuators) are required to describe the experiment to be simulated.

3.2 Vacuum prestressing simulation

In order to simulate the vacuum prestressing of Vacuumatics a so-called 'Atmospheric Pressure Model' is developed by Habanera. This Model is based on the same basic principles as explained in chapter 2 and simulates the prestressing forces on each individual edge particle from an imaginative covering skin (as the flexible enclosing is not physically modelled). Considering the fact that the functionality and comprehensiveness of the simulation are aided by relatively simple equations, the indirect vacuum prestressing force of each particle is represented by a pair of skin forces at the inflections points (as shown in Fig.2b).

To demonstrate the simulation process of HADES a brief outline of the basic steps. First, a predetermined amount of circular elements ('grains') is generated and placed in a (not necessarily) regular fashion. Secondly, given an initial skin curvature, the Atmospheric Pressure Model searches the edge particles that are (partly) covered and are mutually interconnected by the skin that forms a closed line (or surface in 3D) to encapsulate all particles. When this fictitious skin is generated and the points of inflection are located on the edge particles, the individual prestress forces are calculated and applied to the particles. As a result, the system of particles is no longer in balance, leading to a compaction of the particle packing. The so-called Hertz Contact Model, also active in the simulation, accounts for the contact forces to prevent interpenetration. As a result, the simulation will continue until a stable situation is reached when the external forces, such as the applied atmospheric pressure (and possible gravitational forces) are in balance with the internal contact forces. Eventually, an output file can be generated, containing data on the vacuum prestressing forces, the particle coordinates and the contact forces at every (simulation) time step. By means of a data plotting program (like Gnuplot or MATLAB) the data generated by HADES can be visualised (Fig.4 and 5).

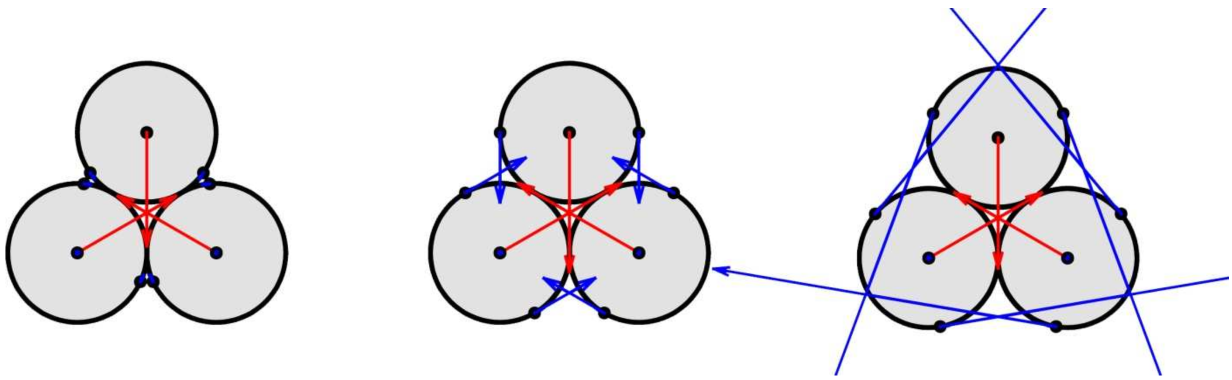

Fig. 4: plotted vacuum prestressing forces with $R_s = 0.1*R_p$ (a), $R_s = R_p$ (b) and $Rs = 5*R_p$ (c)

In Fig.4a b and c, the blue vectors represent the skin forces whereas the red vectors represent the direct vacuum prestressing forces. The amount of vacuum pressure is kept constant throughout the simulation. From these figures, it is clearly illustrated that a decrease in skin coverage (and thus an increase in the radii of the skin curvature) results in an increase in skin forces (and thus indirect prestressing forces) and a decrease in direct prestressing forces. The length of each vectors illustrates the relative magnitude of these forces. In Fig.5 the resulting contact forces ($F_{n;i}$) are represented by grid lines connecting the centre points of the particles, where the width of these lines illustrate the mutual strength of these forces. The related variables are depicted in the corresponding table (Table1).

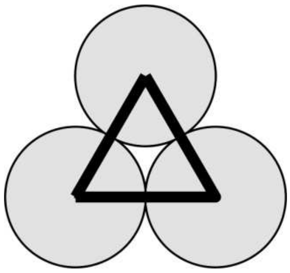

Fig. 5: plotted contact forces

Table1: variables (when $R_{p;i}$ = 50mm and P_v = 0.1MPa)

$R_{s;i}$ [mm]	5	50	250
α_i [rad]	4.4	π	2.4
$F_{v;dir;i}$ [N/mm^1]	8.2	10.0	9.4
$F_{s;i}$ [N/mm^1]	0.5	5.0	25.0
$F_{v;indir;i}$ [N/mm^1]	0.8	10.0	46.9
$F_{n;i}$ [N/mm^1]	5.2	11.6	32.5

3.3 Beam bending simulation

In the previous section we have seen that the vacuum prestressing of Vacuumatics can be simulated by means of Discrete Element Modelling. As the exact position of every particle is constantly reanalysed, it is possible to calculate the deflection and the bending forces of a beam-shaped vacuumatic structure subjected to a bending moment. Fig.6 illustrate a simplified representation of a simply supported beam-shaped vacuumatic structure, depicting the vacuum prestressing forces (analogue to the examples in the previous section) due to the vacuum prestressing (Fig.6a) as well as due to an additional external load at midspan (Fig.6b).

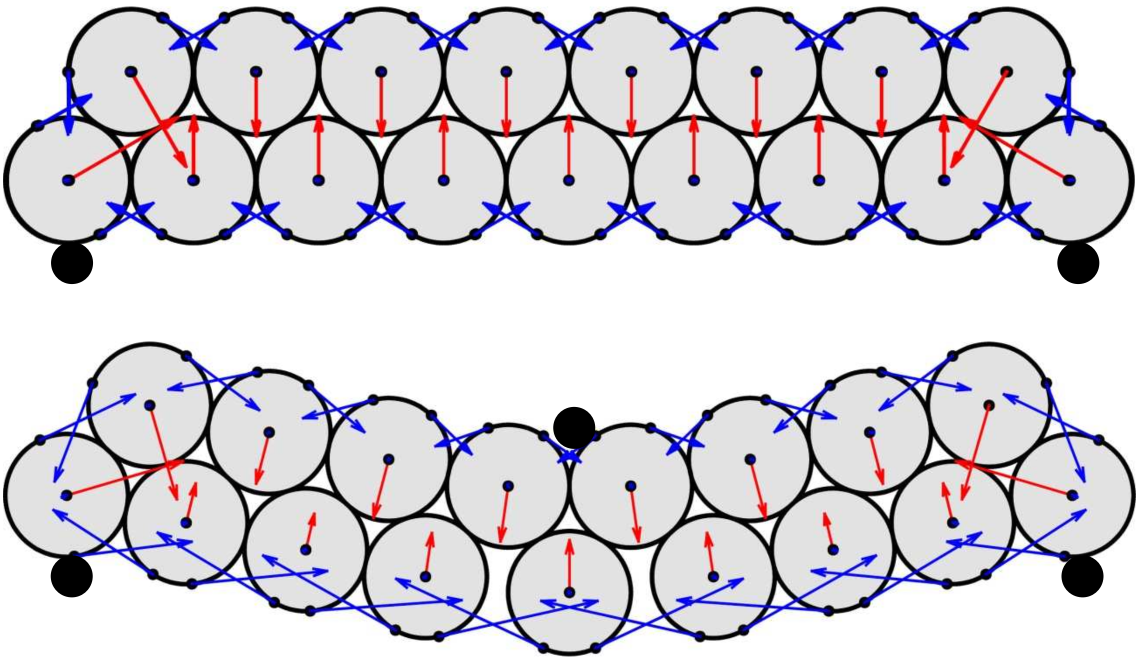

Fig. 6: plotted vacuum prestressing forces in vacuumatic state (a) and under additional external loading (b)

To raise a note of caution here, the total skin length (perimeter) is calculated by the Atmospheric Pressure Model (in HADES) and kept constant throughout each time step. This implies that when the overall structure tends to deform, for instance due to an external force, the positions of particles may change, possibly leading to a change of the individual skin radii in order to keep the length of the membrane fixed. Although the material properties of the particles can be specifically assigned in HADES, the material aspects of the membrane envelope are therefore not taken into account.

Fig.7 illustrates the force distribution of the same beam-shaped vacuumatic structure, due to the vacuum prestressing (Fig.7a) as well as due to an additional external point load at midspan (Fig.7b). The width (and colour) of each gridline represents the mutual strength of these forces (although not proportionally). As Fig.7b illustrates, the edge particles at the tensile zone of the structure tend to lose contact. This implies that the tensile bending forces exceed the compressive (contact) forces due to the vacuum prestressing. The only way the structure is able to take up the tensile forces, is by tensioning the skin (analogue to the reinforcement bars in reinforced concrete structures). Subsequently it can be stated that the strength and stiffness of Vacuumatics is aided by a strong and non-elastic (but flexible) enclosing membrane. Experimental research confirms this statement [2].

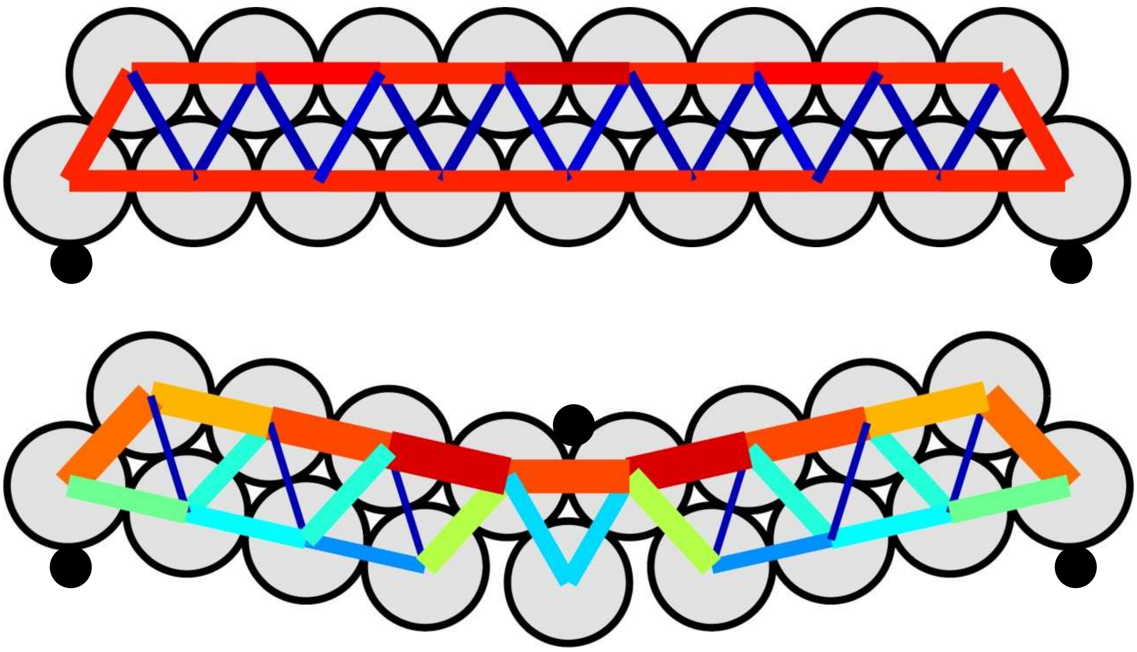

Fig. 7: plotted contact forces in initial vacuumatic state (a) and under additional external loading (b)

4. Discussion / Future work

In order to simulate (and eventually predict) the bending behaviour of Vacuumatics in a realistic manner, a certain type of 'Skin Model' is required in order to incorporate the characteristics of the skin material. Attempts have been made to enhance the existing Atmospheric Pressure Model. However, this approach was found to be immensely complex due to complexity of the equations needed to determine the skin radius according to the elongation and elasticity of the membrane. Furthermore, various additional aspects, like friction between skin and (edge) particles, need to be incorporated. In order to benefit the functionality and comprehensiveness of the simulation process, Habanera is currently developing (in close collaboration with the authors) an advanced HADES Model that might be best explained as the 'bead-chain method'. With this approach the skin envelope is physically modelled and represented by a continuous string of (small) particles onto which the vacuum pressure is acting. This 'bead-chain' therefore realistically simulates the actual skin envelope, including its (material) properties and its interaction with the (edge) particles.

5. Conclusions

Discrete Element Modelling can effectively be used to simulate and analyse the structural behaviour of Vacuumatics in general. As analytically determined, the vacuum prestressing of vacuumatic structures can be described by two separate prestressing components, referred to as the direct prestressing force and the indirect prestressing force, acting on each individual edge particle. By means of a specialised Atmospheric Pressure Model these prestressing forces can be simulated in HADES (by Habanera). From practical point of view the indirect prestressing component can be best simulated by the so-called skin forces.

The DEM simulations provide us with insight into the force distribution and particle displacements of vacuumatic structures due to vacuumatic prestressing as well as additional external forces. Therefore, the bending behaviour of beam-shaped Vacuumatics can be analysed in detail (graphically as well as numerically). As a result, these simulations enable us to predict the structural behaviour of vacuumatic structures in general. An enhanced approach is needed, however, in order to take into account the material aspects of the enclosing membrane. A new HADES Model is currently being developed.

6. References

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